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STOCHASTIC MODELLING OF DEFORESTATION EFFECT IN NIGERIA: A COMPARATIVE STUDY

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Abstract: Deforestation is a key problem for many countries including Nigeria. About 3500 to 4000 square Kilometre of the Nigeria's land is lost to deforestation each year which calls for an urgent attention. An earlier study used a stochastic model known as the Interval Transition probability of a Semi-Markov Process to study the long-run effect of deforestation on Nigeria's total land area with the results: Nigeria will loss 0.042, 0.272, 0.383, 0.408, and 0.409% of her total land area in the 20th, 200th, 500th, 1000th and 1120th years, respectively beginning from 2017. The limitation of this study is that in computing the model parameters, a history of data for land lost due to deforestation in Nigeria is needed which is not readily available. Hence, the motivation for this research. In this study, a modified exponential distribution is proposed to study the long run effect of deforestation on the Nigeria's total land area that does not depend on the history of data but on the current information. The result indicates that 7.783, 55.525, 86.809, 98.260 and 98.930% of the Nigeria's land will be lost to deforestation in the 20th, 200th, 500th, 1000th and 1120th, respectively from 2017. Agreeing with the result of the earlier study is that, if nothing is done to stop or control deforestation, Nigeria will loss almost her entire land area in about 1120 years from 2017. The modified exponential distribution is recommended since its prediction depends on the current information and not on the history.

Keywords: Deforestation, exponential distribution, semi-Markov model, stochastic modelling, Weibull distribution

Introduction

Deforestation results from the removal of trees without sufficient replacement, which leads to reduction in habitat, biodiversity as well as wood and quality of life. Deforestation can denote activities of clearing the forest for fuel, commercial purpose etc (Mfon Jr. *et al*., 2014; Kwaghkor *et al*., 2018). Forests are not only one of the most valuable natural resources but also crucial to ensure decent living conditions for all plants, animals, and humans. The rate at which our planet is being stripped of its forest cover, however, is alarming (Makki, 2009). According to the United Nation's Food and Agriculture Organization (FAO), over 30 million acres (or 13 million hectares) of forests are destroyed by human activity every year worldwide, while Olasupo (2016) reported that Nigeria loses about 3500 to 4000 Square Kilometre of land yearly to deforestation.

Several studies have modelled and are still trying to Model deforestation effects. However, the application of these models are constrained in developing countries (like Nigeria) by the lack of standard ecological and socio-economic data (Lambin, 1994). Kwaghkor *et al*. (2018) used the Interval Transition Probability of Semi – Markov process(where the stay in a state before transition is described by a probability distribution: in this case a Weibull distribution) proposed by Howard (1971) to give a good prediction of the effect of deforestation on a long-run if the rate of deforestation at some constant intervals (yearly) is known. The limitation of their study is that in computing the parameters of the Weibull distribution, a history of data for land lost due to deforestation in Nigeria is needed which is not readily available. This is the motivation for this research. In this study, a modified exponential distribution is proposed to study the long-run effect of deforestation on the Nigeria's total land area that does not depend on the history of land lost due to deforestation but on the current information.

The remaining part of this research work shall be discussed under the following sub-heading: The Model, Results, Discussion and Conclusion.

The Model

Here, the model equation based on the Weibull distribution and the modified exponential distribution model, the analysis and comparison of the results will be presented and discussed.

The interval transition probability from state *i* to state *j* in the interval $(0, t)$: The model equation

$\Phi_{ij}(t)$, defined as the probability that the process will occupy state j at time t if it entered state i at time zero (called the interval transition probability from state i to state j in the interval $(0, t)$) is given by Howard (1971) as

$$\Phi_{ij}(t) = \delta_{ij}\overline{w_i}(t) + \sum_{k=0}^{N} P_{ik}\int_0^t h_{ik}(m)\Phi_{kj}(t-m)dm \qquad (1)$$

$$\delta_{ij} = \begin{cases}1, & i = j\\ 0, & i \neq j\end{cases} \qquad i, j = 1,2,3,\ldots$$

Equation (1) is called the Interval Transition Probability from state i to state j in the interval $(0, t)$ assuming that the process could have made its first transition from state i to some state k at a time m, $0 < m \leq t$, and then by some succession of transitions have made its way to state j at time t. The quantity δ_{ij} ensures that the term in which it appears occurs only when $i = j$.

In practical sense, $\Phi_{ij}(t)$ is referred as the effect of deforestation at a particular time t (in years) and P_{ik} is the average rate of land lost per year due to deforestation (Kwaghkor *et al*., 2018).

The transition process from forestation

A possible transition from forestation as a result of deforestation is presented in Fig. 1.

Fig. 1: Effects of deforestation

p_{12}: the average rate of Nigeria land lost due to deforestation per year

Following from Fig. 1 above, equation (1) becomes

$$\Phi_{12}(t) = \delta_{12}\overline{w_1}(t) + \sum_{k=1}^{2} P_{1k}\int_0^t h_{1k}(m)dm\Phi_{k2}(t-m) \qquad (2)$$

Since the process is moving from state 1 to state 2 without branching to any state, $k = 2, m = t$ and $\delta_{12} = 0$, equation (2) now becomes

$$\Phi_{12}(t) = P_{12}\int_0^t h_{12}(n)dn \qquad (3)$$

Based on the assumption that the process follows a Weibull Distribution, equation (3) now becomes

$$\Phi_{12}(t) = P_{12}\left[1 - \exp\left(-\frac{t}{\alpha}\right)^{\beta}\right] \qquad (4)$$

Equation (4) describes the effect of deforestation at a particular time t (in years) and P_{12} is the average rate of land lost per year due to deforestation (Kwaghkor *et al*., 2018).

Estimation of parameters

The parameters α and β are obtained below using (Dubey, 1967; Marks, 2005; Kwaghkor *et al*., 2018).

$$\beta = \frac{\ln[-\ln(1-p_1)] - \ln[-\ln(1-p_2)]}{\ln x_{p_1} - \ln x_{p_2}} \qquad (5)$$

and

$$\alpha = exp\left[\ln x_{p_1} - \ln(-\ln(1-p_1))/\beta\right] \qquad (6)$$

or

$$\alpha = exp\left[\ln x_{p_2} - \ln(-\ln(1-p_2))/\beta\right] \qquad (7)$$

or

$$\alpha = exp\left[\frac{1}{2}\sum_{i=1}^{2}\{\ln x_{p_i} - \ln(-\ln(1-p_i))/\beta\}\right] \qquad (8)$$

Note that the estimation of β and α only requires two percentiles from a sample. Kwaghkor *et al*., 2018 used the 25th and the 75th percentiles in their work.

The modified exponential distribution model

This modified exponential distribution was first proposed by Kwaghkor *et al*. (2019) to study long term unemployment rate. The modified exponential distribution was presented as

$$f(t;\lambda) = \begin{cases}(1-\exp(-J/n))\exp(-(1-\exp(-J/n))t), & t > 0\\ 0, & \text{otherwise}\end{cases} \qquad (9)$$

where $\lambda = 1 - \exp(-J/n)$, J =Number of persons entering unemployment state at present and n = Number of persons from the source of entering unemployment state at present.

The Cumulative Distribution Function (CDF) of the modified exponential distribution written as the probability of lifetime being less than some value, t, is

$$F(t;\lambda) = P(T \le t) = 1 - exp[-(1-\exp(-J/n))t] \qquad (10)$$

Equation (10) was used to give a probable unemployment rate if J (Number of persons entering unemployment state at present) and n (Number of persons from the source of entering unemployment state at present) are known.

In this study, J and n are redefined as follows

J =Average land lost per year due to deforestation in Nigeria, and

n = Total land area in Nigeria.

Results and Discussion

Results based on the interval transition probability from state i to state j in the interval $(\mathbf{0}, \boldsymbol{t})$

The 25th and 75th percentiles using the data in Table 1 and TheWorldFactbook (2016); the values of parameters of the Weibull distribution using equations (5) and (6) are presented in Table 2.

Table 1: Average land lost in Nigeria per year due to deforestation (formed from Olasupo (2016))

Year	Land lost (Km^2)
1	3520
2	3600
3	4000
4	3900
5	3700
6	3620
7	2710
8	3810
9	3850
10	3540
11	4000
12	3910
13	3820
14	3630
15	3650
16	3750
17	3920
18	3900
19	3990
20	3770

Total land area in Nigeria is 923,768 Km^2 (TheWorldFactbook, 2016).

Table 2: Parameter values of the model for deforestation

Parameter	Value
p_1	0.25
p_2	0.75
x_{p_1}	3625
x_{p_2}	3905
p_{12}	0.0041
β	21
α	3845

The Table generated from fitting the values of Table 2 to equation (4) is presented in Table 4 and the graph obtained from the table is presented in Fig. 2.

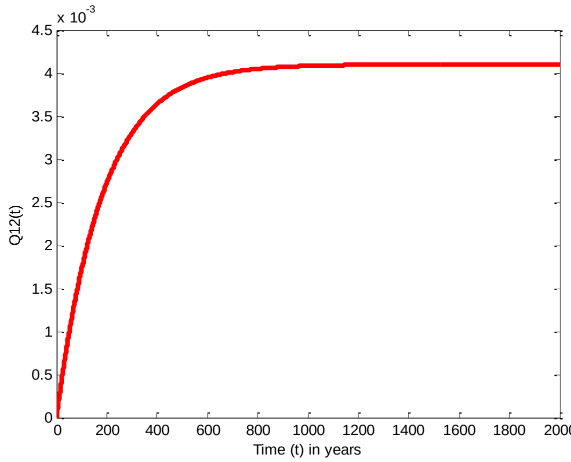

Fig. 2: Graph of interval transition probabilities from forestation

Result based on the modified exponential distribution

From Olasupo (2016) and TheWorldFactbook (2016); the parameter values for the modified exponential distribution are given in Table 3.

Table 3: Parameter values for deforestation using the modified exponential distribution

Parameter	Value	Source
J	3,750 km^2	Olasupo (2016)
n	923,768km^2	TheWorldFactbook (2016)

The Table generated from fitting the values of Table 3 to equation (10) is presented in Table 5 and the graph obtained from the Table is presented in Fig. 3.

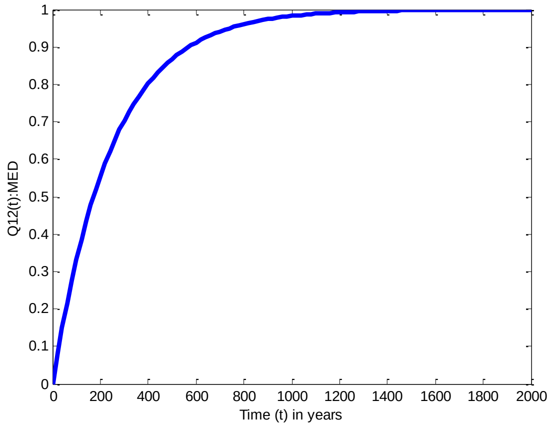

Fig. 3: Effect of deforestation to Nigeria's land area by the modified exponential distribution

Table 4 shows the interval transition probabilities from forestation due to deforestation using the Interval Transition probability of a Semi-Markov model whose stay in the state is described by the Weibull distribution. This is describing the gradual loss of Nigeria's land due to deforestation. Converting the result to percentage, it can be seen that Nigeria will loss 0.042, 0.272, 0.383, 0.408, and 0.409% in the 20th, 200th, 500th, 100th and 1120th years, respectively beginning from 2017. It can also be seen from both Table 4 and Fig. 2 that from the 1120th year upward, the interval transition probabilities converges very slowly which can be negligible from the figure. The slowness is due to the nature of the Weibull Distribution. This result simply indicates that if Nigeria continues to loss about $3500 - 4000\ km^2$ of her land each year (due to deforestation); it will take about 1120 years to loss her total land area completely if nothing is done.

Table 4: Values of interval transition probability from forestation

t	$\Phi_{12}(t)$
0	0
20	0.000424260
40	0.000804620
60	0.001145620
80	0.001451334
100	0.001725413
120	0.001971132
140	0.002191423
160	0.002388919
180	0.002565979
200	0.002724717
220	0.002867029
240	0.002994614
260	0.003108998
280	0.003211545
300	0.003303481
320	0.003385903
340	0.003459797
360	0.003526044
380	0.003585436
400	0.003638682
420	0.003686418
440	0.003729215
460	0.003767583
480	0.003801981
500	0.003832819
520	0.003860467
540	0.003885253
560	0.003907475
580	0.003927397
600	0.003945258
620	0.003961270
640	0.003975625
660	0.003988495
680	0.004000034
700	0.004010378
720	0.004019652
740	0.004027966
760	0.004035420
780	0.004042103
800	0.004048094
820	0.004053465
840	0.004058280
860	0.004062597
880	0.004066467
900	0.004069937
920	0.004073048
940	0.004075837
960	0.004078337
980	0.004080579
1000	0.004082588
1020	0.004084390
1040	0.004086005
1060	0.004087453
1080	0.004088752
1100	0.004089916
1120	0.004090959
1140	0.004091895
1160	0.004092733
1180	0.004093485
1200	0.004094159
1220	0.004094764
1240	0.004095305
1260	0.004095791
1280	0.004096227
1300	0.004096617
1320	0.004096967
1340	0.004097281
1360	0.004097562
1380	0.004097814
1400	0.004098040
1420	0.004098243
1440	0.004098425
1460	0.004098588
1480	0.004098734
1500	0.004098865
1520	0.004098982
1540	0.004099088
1560	0.004099182
1580	0.004099267
1600	0.004099342
1620	0.004099410
1640	0.004099471
1660	0.004099526
1680	0.004099575
1700	0.004099619
1720	0.004099658
1740	0.004099694
1760	0.004099725
1780	0.004099754
1800	0.004099779
1820	0.004099802
1840	0.004099822
1860	0.004099841
1880	0.004099857
1900	0.004099872
1920	0.004099885
1940	0.004099897
1960	0.004099908
1980	0.004099917
2000	0.004099926

Table 5: Values of the effect of deforestation based on the modified exponential distribution

t	$\Phi(t)_{12}$	% land loss	Land loss (km^2)
0	0	0	0
20	0.077829037	7.7829037	71895.97
40	0.149600716	14.9600716	138196.4
60	0.215786474	21.5786474	199336.6
80	0.276821058	27.6821058	255718.4
100	0.333105379	33.3105379	307712.1
120	0.385009145	38.50091451	355659.1
140	0.432873291	43.28732913	399874.5
160	0.477012217	47.70122172	440648.6
180	0.517715853	51.7715853	478249.3
200	0.555251564	55.52515639	512923.6
220	0.589865907	58.98659066	544899.2
240	0.621786248	62.17862483	574386.2
260	0.651222261	65.12222606	601578.3
280	0.678367296	67.83672963	626654
300	0.70339966	70.33996601	649778.1
320	0.726483779	72.64837791	671102.5
340	0.747771283	74.77712833	690767.2
360	0.767402002	76.74020015	708901.4
380	0.78550488	78.55048798	725624.3
400	0.802198829	80.21988286	741045.6
420	0.817593503	81.75935034	755266.7
440	0.831790025	83.17900254	768381
460	0.844881646	84.48816459	780474.6
480	0.856954358	85.6954358	791627
500	0.868087463	86.80874627	801911.4
520	0.878354088	87.83540885	811395.4
540	0.887821673	88.78216727	820141.3
560	0.896552404	89.65524039	828206.4
580	0.904603631	90.46036308	835643.9
600	0.912028238	91.20282384	842502.5
620	0.918874996	91.88749959	848827.3
640	0.925188877	92.51888769	854659.9
660	0.931011355	93.10113546	860038.5
680	0.936380674	93.63806745	864998.5
700	0.941332105	94.13321053	869572.5
720	0.945898171	94.58981711	873790.5
740	0.950108864	95.01088644	877680.2
760	0.953991843	95.39918434	881267.1
780	0.957572614	95.7572614	884574.9
800	0.960874697	96.08746966	887625.3
820	0.963919781	96.39197813	890438.2
840	0.96672787	96.672787	893032.3
860	0.969317408	96.93174078	895424.4
880	0.971705404	97.17054045	897630.4
900	0.973907546	97.39075456	899664.6
920	0.975938296	97.59382962	901540.6
940	0.977810995	97.78109955	903270.5
960	0.979537944	97.95379443	904865.8
980	0.981130486	98.11304864	906336.9
1000	0.982599082	98.25990825	907693.6
1020	0.983953379	98.39533792	908944.6
1040	0.985202272	98.52022722	910098.3
1060	0.986353965	98.63539651	911162.2
1080	0.987416023	98.74160229	912143.3
1100	0.988395422	98.83954217	913048.1
1120	0.989298595	98.92985949	913882.4
1140	0.990131475	99.01314749	914651.8
1160	0.990899533	99.08995327	915361.3
1180	0.991607813	99.16078133	916015.6
1200	0.992260969	99.22609692	916618.9
1220	0.99286329	99.28632905	917175.3
1240	0.993418734	99.34187337	917688.4
1260	0.993930947	99.39309473	918161.6
1280	0.994403296	99.44032959	918597.9
1300	0.994838882	99.4838882	919000.3
1320	0.995240567	99.52405668	919371.4
1340	0.995610989	99.56109889	919713.6
1360	0.995952581	99.59525814	920029.1
1380	0.996267588	99.62675881	920320.1
1400	0.996558078	99.65580781	920588.5
1420	0.99682596	99.68259596	920835.9
1440	0.997072992	99.70729921	921064.1
1460	0.997300798	99.73007983	921274.6
1480	0.997510875	99.75108746	921468.6
1500	0.997704601	99.77046008	921647.6
1520	0.99788325	99.78832495	921812.6
1540	0.998047994	99.80479942	921964.8
1560	0.998199917	99.81999169	922105.1
1580	0.998340016	99.83400156	922234.6
1600	0.998469211	99.84692106	922353.9
1620	0.99858835	99.85883505	922464
1640	0.998698218	99.86982178	922565.5
1660	0.998799534	99.87995343	922659
1680	0.998892965	99.88929654	922745.4
1700	0.998979125	99.89791248	922824.9
1720	0.999058579	99.90585785	922898.3
1740	0.999131848	99.91318485	922966
1760	0.999199416	99.91994159	923028.4
1780	0.999261725	99.92617246	923086
1800	0.999319184	99.93191838	923139.1
1820	0.999372171	99.93721711	923188
1840	0.999421034	99.94210344	923233.2
1860	0.999466095	99.94660947	923274.8
1880	0.999507648	99.95076481	923313.2
1900	0.999545967	99.95459674	923348.6
1920	0.999581304	99.95813043	923381.2
1940	0.999613891	99.9613891	923411.3
1960	0.999643941	99.96439415	923439.1
1980	0.999671653	99.96716532	923464.7
2000	0.999697208	99.96972081	923488.3

Figure 3 and Table 5 are showing the transition probabilities of moving from forestation due to deforestation using the modified exponential distribution. This is also describing the gradual loss of Nigeria's land due to deforestation. Converting the transition probabilities of Table 5 to percentage, it can be seen that Nigeria will loss about 7.783%, (71895.97 km^2) 55.525% (512923.60 km^2), 86.809% (801911.40 km^2), 98.260% (907693.60 km^2) and 98.930% (913882.40 km^2) of her land to deforestation in the 20th, 200th, 500th, 1000th and 1120th, respectively beginning from 2017. Agreeing with the result of Kwaghkor *et al.* (2018) is that if nothing is done to stop or control deforestation, Nigeria may loss almost her entire land area in about 1120 years. Also to note is that the percentage loss is moving gradually from zero percent to hundred percent by the modified exponential distribution which seem to be more realistic in nature than the result based on the Interval Transition probability: making the modified exponential distribution a more better model for studying decay processes in real life situations. The modified exponential distribution is also recommended since its prediction depends on the current information and not on the history.

Conclusion

In this paper, a modified exponential distribution was proposed to study the effect of deforestation on the Nigeria's land area. The aim of the paper is to compare the result with that of an earlier work by Kwaghkor *et al*. (2018) which uses the interval transition probability of a Semi-Markov model to study the effect of deforestation on the Nigeria's land. The results agreed in the fact that if nothing is done to stop or reduce deforestation, Nigeria will loss almost her entire land area in about 1120 years. Also to note is that the percentage loss is moving gradually from zero percent to hundred percent by the modified exponential distribution which seem to be more realistic in nature than the result based on the Interval Transition probability of a Semi-Markov model: making the modified exponential distribution a more better model for studying decay processes in real life situations.

Conflict of Interest

Author has declared that there is no conflict of interest reported in this work.

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